SUGGESTIONS FOR FURTHER READING


QUESTIONS

1. A particle is moving at a speed of less than \( c/2 \). If the speed of the particle is doubled, what happens to its momentum?
2. Give a physical argument showing that it is impossible to accelerate an object of mass \( m \) to the speed of light, even with a continuous force acting on it.
3. The upper limit of the speed of an electron is the speed of light, \( c \). Does that mean that the momentum of the electron has an upper limit?
4. Because mass is a measure of energy, can we conclude that the mass of a compressed spring is greater than the mass of the same spring when it is not compressed?
5. Photons of light have zero mass. How is it possible that they have momentum?
6. “Newtonian mechanics correctly describes objects moving at ordinary speeds, and relativistic mechanics correctly describes objects moving very fast.” “Relativistic mechanics must make a smooth transition as it reduces to Newtonian mechanics in a case where the speed of an object becomes small compared to the speed of light.” Argue for or against each of these two statements.
7. Two objects are identical except that one is hotter than the other. Compare how they respond to identical forces.

8. With regard to reference frames, how does general relativity differ from special relativity?
9. Two identical clocks are in the same house, one upstairs in a bedroom, and the other downstairs in the kitchen. Which clock runs more slowly? Explain.
10. A thought experiment. Imagine ants living on a merry-go-round, which is their two-dimensional world. From measurements on small circles they are thoroughly familiar with the number \( \pi \). When they measure the circumference of their world, and divide it by the diameter, they expect to calculate the number \( \pi = 3.14159 \ldots \). We see the merry-go-round turning at relativistic speed. From our point of view, the ants’ measuring rods on the circumference are experiencing Lorentz contraction in the tangential direction; hence the ants will need some extra rods to fill that entire distance. The rods measuring the diameter, however, do not contract, because their motion is perpendicular to their lengths. As a result, the computed ratio does not agree with the number \( \pi \). If you were an ant, you would say that the rest of the universe is spinning in circles, and your disk is stationary. What possible explanation can you then give for the discrepancy, in view of the general theory of relativity?

PROBLEMS

2.1 Relativistic Momentum and the Relativistic Form of Newton’s Laws

1. Calculate the momentum of a proton moving with a speed of (a) 0.010\( c \), (b) 0.50\( c \), (c) 0.90\( c \). (d) Convert the answers of (a)–(c) to MeV/\( c \).
2. An electron has a momentum that is 90% larger than its classical momentum. (a) Find the speed of the electron. (b) How would your result change if the particle were a proton?
3. Consider the relativistic form of Newton’s second law. Show that when \( F \) is parallel to \( \mathbf{v} \),

\[
F = m \left( 1 - \frac{v^2}{c^2} \right)^{-3/2} \frac{dv}{dt}
\]

where \( m \) is the mass of an object and \( v \) is its speed.
4. A charged particle moves along a straight line in a uniform electric field \( E \) with a speed \( v \). If the motion and the electric field are both in the \( x \) direction, (a) show
that the magnitude of the acceleration of the charge \( q \) is given by
\[
a = \frac{dq}{dt} = \frac{qE}{m} \left( 1 - \frac{v^2}{c^2} \right)^{3/2}
\]
(b) Discuss the significance of the dependence of the acceleration on the speed. (c) If the particle starts from rest at \( x = 0 \) at \( t = 0 \), find the speed of the particle and its position after a time \( t \) has elapsed. Comment on the limiting values of \( v \) and \( x \) as \( t \to \infty \).

5. Recall that the magnetic force on a charge \( q \) moving with velocity \( v \) in a magnetic field \( B \) is equal to \( qv \times B \). If a charged particle moves in a circular orbit with a fixed speed \( v \) in the presence of a constant magnetic field, use the relativistic form of Newton’s second law to show that the frequency of its orbital motion is
\[
f = \frac{qB}{2\pi m} \left( 1 - \frac{v^2}{c^2} \right)^{1/2}
\]
6. Show that the momentum of a particle having charge \( e \) moving in a circle of radius \( R \) in a magnetic field \( B \) is given by \( p = 300BR \), where \( p \) is in MeV/c, \( B \) is in teslas, and \( R \) is in meters.

2.2 Relativistic Energy
7. Show that the energy–momentum relationship given by \( E^2 = p^2c^2 + (mc^2)^2 \) follows from the expressions \( E = \gamma mc^2 \) and \( p = \gamma mu \).
8. A proton moves at a speed of 0.95c. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.
9. An electron has a kinetic energy 5 times greater than its rest energy. Find (a) its total energy and (b) its speed.
10. Find the speed of a particle whose total energy is 50% greater than its rest energy.
11. A proton in a high-energy accelerator is given a kinetic energy of 50 GeV. Determine the (a) momentum and (b) speed of the proton.
12. An electron has a speed of 0.75c. Find the speed of a proton that has (a) the same kinetic energy as the electron and (b) the same momentum as the electron.
13. Protons in an accelerator at the Fermi National Laboratory near Chicago are accelerated to an energy of 400 times their rest energy. (a) What is the speed of these protons? (b) What is their kinetic energy in MeV?
14. How long will the Sun shine, Nellie? The Sun radiates about \( 4.0 \times 10^{26} \) J of energy into space each second. (a) How much mass is released as radiation each second? (b) If the mass of the Sun is \( 2.0 \times 10^{30} \) kg, how long can the Sun survive if the energy release continues at the present rate?
15. Electrons in projection television sets are accelerated through a total potential difference of 50,000 V. (a) Calculate the speed of the electrons using the relativistic form of kinetic energy assuming the electrons start from rest. (b) Calculate the speed of the electrons using the classical form of kinetic energy. (c) Is the difference in speed significant in the design of this set in your opinion?
16. As noted in Section 2.2, the quantity \( E - p^2c^2 \) is an invariant in relativity theory. This means that the quantity \( E^2 - p^2c^2 \) has the same value in all inertial frames even though \( E \) and \( p \) have different values in different frames. Show this explicitly by considering the following case. A particle of mass \( m \) is moving in the +x direction with speed \( u \) and has momentum \( p \) and energy \( E \) in the frame \( S \). (a) If \( S' \) is moving at speed \( v \) in the standard way, find the momentum \( p' \) and energy \( E' \) observed in \( S' \). (Hint: Use the Lorentz velocity transformation to find \( p' \) and \( E' \). Does \( E = E' \) and \( p = p' \)?) (b) Show that \( E^2 - p^2c^2 \) is equal to \( E'^2 - p'^2c^2 \).

2.3 Mass as a Measure of Energy
17. A radium isotope decays to a radon isotope, \(^{226}\text{Rn}\), by emitting an alpha particle (a helium nucleus) according to the decay scheme \(^{226}\text{Ra} \rightarrow ^{222}\text{Rn} + ^{4}\text{He} \). The masses of the atoms are 226.0254 (Ra), 222.0175 (Rn), and 4.0026 (He). How much energy is released as the result of this decay?
18. Consider the decay \(^{55}\text{Cr} \rightarrow ^{55}\text{Mn} + e^- \), where \( e^- \) is an electron. The \(^{55}\text{Cr} \) nucleus has a mass of 54.9279 u, and the \(^{55}\text{Mn} \) nucleus has a mass of 54.9244 u. (a) Calculate the mass difference in MeV. (b) What is the maximum kinetic energy of the emitted electron?
19. Calculate the binding energy in MeV per nucleon in the isotope \(^{12}\text{C} \). Note that the mass of this isotope is exactly 12 u, and the masses of the proton and neutron are 1.007276 u and 1.008665 u, respectively.
20. The free neutron is known to decay into a proton, an electron, and an antineutrino \( \bar{\nu} \) (of negligible rest mass) according to
\[ n \rightarrow p + e^- + \bar{\nu} \]
This is called beta decay and will be discussed further in Chapter 13. The decay products are measured to have a total kinetic energy of 0.781 MeV ± 0.005 MeV. Show that this observation is consistent with the excess energy predicted by the Einstein mass–energy relationship.

2.4 Conservation of Relativistic Momentum and Energy
21. An electron having kinetic energy \( K = 1.000 \) MeV makes a head-on collision with a positron at rest. (A positron is an antimatter particle that has the same mass as the electron but opposite charge.) In the collision the two particles annihilate each other and are replaced by two gamma rays of equal energy, each traveling at equal angles \( \theta \) with the electron’s direction of motion. (Gamma rays are massless particles of ele-