tem. (c) How much energy would be carried away in a 1-quantum change?

4. (a) Use Stefan's law to calculate the total power radiated per unit area by a tungsten filament at a temperature of 3000 K. (Assume that the filament is an ideal radiator.) (b) If the tungsten filament of a lightbulb is rated at 75 W, what is the surface area of the filament? (Assume that the main energy loss is due to radiation.)
5. Consider the problem of the distribution of blackbody radiation described in Figure 3.3. Note that as T *increases*, the wavelength $\lambda_{\max}$ at which $u(\lambda, T)$ reaches a maximum shifts toward shorter wavelengths. (a) Show that there is a general relationship between temperature and $\lambda_{\max}$ stating that $T\lambda_{\max} =$ constant (Wien's displacement law). (b) Obtain a numerical value for this constant. (*Hint:* Start with Planck's radiation law and note that the slope of $u(\lambda, T)$ is zero when $\lambda = \lambda_{\max}$.)
6. *Planck's fundamental constant, h.* Planck ultimately realized the great and fundamental importance of h, which, much more than a curve-fitting parameter, is actually the measure of all quantum phenomena. In fact, Planck suggested using the universal constants h, c (the velocity of light), and G (Newton's gravitational constant) to construct "natural" or universal units of length, time, and mass. He reasoned that the current units of length, time, and mass were based on the accidental size, motion, and mass of our particular planet, but that truly universal units should be based on the quantum theory, the speed of light in a vacuum, and the law of gravitation—all of which hold anywhere in the universe and at all times. Show that the expressions $\left(\frac{hG}{c^3}\right)^{1/2}$, $\left(\frac{hG}{c^5}\right)^{1/2}$, and $\left(\frac{hc}{G}\right)^{1/2}$ have dimensions of length, time, and mass and find their numerical values. These quantities are called, respectively, the Planck length, the Planck time, and the Planck mass. Would you care to speculate on the physical meaning of these quantities?

3.3 Derivation of the Rayleigh–Jeans Law and Planck's Law (Optional)

7. *Density of modes.* The essentials of calculating the number of modes of vibration of waves confined to a cavity may be understood by considering a one-dimensional example. (a) Calculate the number of modes (standing waves of different wavelength) with wavelengths between 2.0 cm and 2.1 cm that can exist on a string with fixed ends that is 2 m long. (*Hint:* use $n(\lambda/2) = L$, where $n = 1, 2, 3, 4, 5 \ldots$. Note that a specific value of n defines a specific mode or standing wave with different wavelength.) (b) Calculate, in analogy to our three-dimensional calculation, the number of modes per unit wavelength per unit length, $\frac{\Delta n}{L\,\Delta\lambda}$. (c) Show that in general the number of modes per unit wavelength per unit length for a string of length L is given by

$$\frac{1}{L}\left|\frac{dn}{d\lambda}\right| = \frac{2}{\lambda^2}$$

Does this expression yield the same numerical answer as found in (a)? (d) Under what conditions is it justified to replace $\left|\left(\frac{\Delta n}{L\,\Delta\lambda}\right)\right|$ with $\left|\left(\frac{dn}{L\,d\lambda}\right)\right|$? Is the expression $n = 2L/\lambda$ a continuous function?

3.4 Light Quantization and the Photoelectric Effect

8. Calculate the energy of a photon whose frequency is (a) 5×10^{14} Hz, (b) 10 GHz, (c) 30 MHz. Express your answers in electron volts.
9. Determine the corresponding wavelengths for the photons described in Problem 8.
10. An FM radio transmitter has a power output of 100 kW and operates at a frequency of 94 MHz. How many photons per second does the transmitter emit?
11. The average power generated by the Sun has the value 3.74×10^{26} W. Assuming the average wavelength of the Sun's radiation to be 500 nm, find the number of photons emitted by the Sun in 1 s.
12. A sodium-vapor lamp has a power output of 10 W. Using 589.3 nm as the average wavelength of the source, calculate the number of photons emitted per second.
13. The photocurrent of a photocell is cut off by a retarding potential of 2.92 V for radiation of wavelength 250 nm. Find the work function for the material.
14. The work function for potassium is 2.24 eV. If potassium metal is illuminated with light of wavelength 350 nm, find (a) the maximum kinetic energy of the photoelectrons and (b) the cutoff wavelength.
15. Molybdenum has a work function of 4.2 eV. (a) Find the cutoff wavelength and threshold frequency for the photoelectric effect. (b) Calculate the stopping potential if the incident light has a wavelength of 200 nm.
16. When cesium metal is illuminated with light of wavelength 300 nm, the photoelectrons emitted have a maximum kinetic energy of 2.23 eV. Find (a) the work function of cesium and (b) the stopping potential if the incident light has a wavelength of 400 nm.
17. Consider the metals lithium, beryllium, and mercury, which have work functions of 2.3 eV, 3.9 eV, and 4.5 eV, respectively. If light of wavelength 300 nm is incident on each of these metals, determine (a) which metals exhibit the photoelectric effect and (b) the

maximum kinetic energy for the photoelectron in each case.

18. Light of wavelength 500 nm is incident on a metallic surface. If the stopping potential for the photoelectric effect is 0.45 V, find (a) the maximum energy of the emitted electrons, (b) the work function, and (c) the cutoff wavelength.
19. The active material in a photocell has a work function of 2.00 eV. Under reverse-bias conditions (where the polarity of the battery in Figure 3.14 is reversed), the cutoff wavelength is found to be 350 nm. What is the value of the bias voltage?
20. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. What is the work function of the metal?
21. Figure P3.21 shows the stopping potential versus incident photon frequency for the photoelectric effect for sodium. Use these data points to find (a) the work function, (b) the ratio h/e, and (c) the cutoff wavelength. (d) Find the percent difference between your answer to (b) and the accepted value. (Data taken from R. A. Millikan, *Phys. Rev.*, 7:362, 1916.)

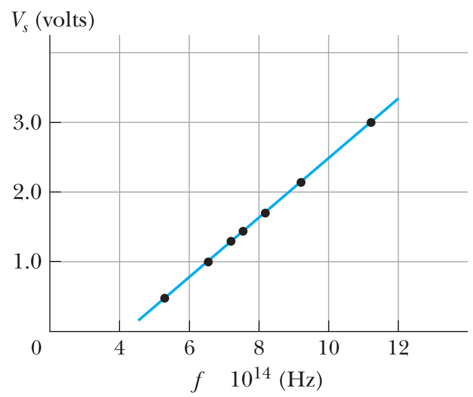

Figure P3.21 Some of Millikan's original data for sodium.

22. Photons of wavelength 450 nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20 cm by a magnetic field whose strength is equal to 2.0×10^{-5} T. What is the work function of the metal?

2.5 The Compton Effect and X-Rays

23. Calculate the energy and momentum of a photon of wavelength 500 nm.
24. X-rays of wavelength 0.200 nm are scattered from a block of carbon. If the scattered radiation is detected at 90° to the incident beam, find (a) the Compton shift, $\Delta\lambda$, and (b) the kinetic energy imparted to the recoiling electron.
25. X-rays with an energy of 300 keV undergo Compton scattering from a target. If the scattered rays are detected at 30° relative to the incident rays, find (a) the Compton shift at this angle, (b) the energy of the scattered x-ray, and (c) the energy of the recoiling electron.
26. X-rays with a wavelength of 0.040 nm undergo Compton scattering. (a) Find the wavelength of photons scattered at angles of 30°, 60°, 90°, 120°, 150°, 180°, and 210°. (b) Find the energy of the scattered electrons corresponding to these scattered x-rays. (c) Which one of the given scattering angles provides the electron with the greatest energy?
27. Show that a photon cannot transfer all of its energy to a free electron. (*Hint:* Note that energy and momentum must be conserved.)
28. In the Compton scattering event illustrated in Figure 3.24, the scattered photon has an energy of 120 keV and the recoiling electron has an energy of 40 keV. Find (a) the wavelength of the incident photon, (b) the angle θ at which the photon is scattered, and (c) the recoil angle ϕ of the electron.
29. Gamma rays (high-energy photons) of energy 1.02 MeV are scattered from electrons that are initially at rest. If the scattering is *symmetric*, that is, if $\theta = \phi$ in Figure 3.24, find (a) the scattering angle θ and (b) the energy of the scattered photons.
30. If the maximum energy given to an electron during Compton scattering is 30 keV, what is the wavelength of the incident photon? (*Hint:* What is the scattering angle for maximum energy transfer?)
31. A photon of initial energy 0.1 MeV undergoes Compton scattering at an angle of 60°. Find (a) the energy of the scattered photon, (b) the recoil kinetic energy of the electron, and (c) the recoil angle of the electron.
32. An excited iron (Fe) nucleus (mass 57 u) decays to its ground state with the emission of a photon. The energy available from this transition is 14.4 keV. (a) By how much is the photon energy reduced from the full 14.4 keV as a result of having to share energy with the recoiling atom? (b) What is the wavelength of the emitted photon?
33. Show that the Compton formula

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

results when expressions for the electron energy (Equation 3.33) and momentum (Equation 3.34) are substituted into the relativistic energy expression,

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$