QUESTIONS

1. The probability density at certain points for a particle in a box is zero, as seen in Figure 6.9. Does this imply that the particle cannot move across these points? Explain.
2. Discuss the relation between the zero-point energy and the uncertainty principle.
3. Consider a square well with one finite wall and one infinite wall. Compare the energy and momentum of a particle trapped in this well to the energy and momentum of an identical particle trapped in an infinite well with the same width.
4. Explain why a wave packet moves with the group velocity rather than with the phase velocity.
5. According to Section 6.2, a free particle can be represented by any number of waveforms, depending on the values chosen for the coefficients $a(k)$. What is the source of this ambiguity, and how is it resolved?
6. Because the Schrödinger equation can be formulated in terms of operators as $[H]\Psi = [E]\Psi$, is it incorrect to conclude from this the operator equivalence $[H] = [E]$?
7. For a particle in a box, the squared momentum p^2 is a sharp observable, but the momentum itself is fuzzy. Explain how this can be so, and how it relates to the classical motion of such a particle.
8. A philosopher once said that "it is necessary for the very existence of science that the same conditions always produce the same results." In view of what has been said in this chapter, present an argument showing that this statement is false. How might the statement be reworded to make it true?

PROBLEMS

6.1 The Born Interpretation

1. Of the functions graphed in Figure P6.1, which are candidates for the Schrödinger wavefunction of an actual physical system? For those that are not, state why they fail to qualify.

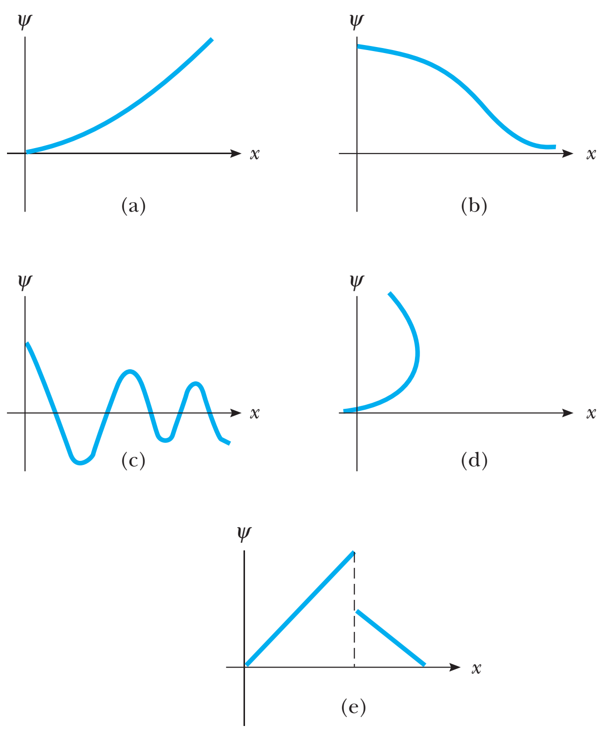

Figure P6.1

2. A particle is described by the wavefunction

$$\psi(x) = \begin{cases} A\cos\left(\dfrac{2\pi x}{L}\right) & \text{for } -\dfrac{L}{4} \le x \le \dfrac{L}{4} \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the normalization constant A. (b) What is the probability that the particle will be found between $x = 0$ and $x = L/8$ if a measurement of its position is made?

6.2 Wavefunction for a Free Particle

3. A free electron has a wavefunction

$$\psi(x) = A\sin(5 \times 10^{10}\, x)$$

where x is measured in meters. Find (a) the electron's de Broglie wavelength, (b) the electron's momentum, and (c) the electron's energy in electron volts.

4. *Spreading of a Gaussian wave packet.* The Gaussian wave packet $\Psi(x, 0)$ of Example 6.3 is built out of plane waves according to the amplitude distribution function $a(k) = (C\alpha/\sqrt{\pi})\exp(-\alpha^2 k^2)$. Calculate $\Psi(x, t)$ for this packet and describe its evolution.

6.3 Wavefunctions in the Presence of Forces

5. In a region of space, a particle with zero energy has a wavefunction

$$\psi(x) = Axe^{-x^2/L^2}$$

(a) Find the potential energy U as a function of x.
(b) Make a sketch of $U(x)$ versus x.

6. The wavefunction of a particle is given by

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

where A, B, and k are constants. Show that ψ is a solution of the Schrödinger equation (Eq. 6.13), assuming the particle is free ($U = 0$), and find the corresponding energy E of the particle.

6.4 The Particle in a Box

7. Show that allowing the state $n = 0$ for a particle in a one-dimensional box violates the uncertainty principle, $\Delta x\, \Delta p \geq \hbar/2$.
8. A bead of mass 5.00 g slides freely on a wire 20.0 cm long. Treating this system as a particle in a one-dimensional box, calculate the value of n corresponding to the state of the bead if it is moving at a speed of 0.100 nm per year (that is, apparently at rest).
9. The nuclear potential that binds protons and neutrons in the nucleus of an atom is often approximated by a square well. Imagine a proton confined in an infinite square well of length 10^{-5} nm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon that is emitted when the proton undergoes a transition from the first excited state ($n = 2$) to the ground state ($n = 1$). In what region of the electromagnetic spectrum does this wavelength belong?
10. An electron is contained in a one-dimensional box of width 0.100 nm. (a) Draw an energy-level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of *all* photons that can be emitted by the electron in making transitions that would eventually get it from the $n = 4$ state to the $n = 1$ state.
11. Consider a particle moving in a one-dimensional box with walls at $x = -L/2$ and $x = L/2$. (a) Write the wavefunctions and probability densities for the states $n = 1$, $n = 2$, and $n = 3$. (b) Sketch the wavefunctions and probability densities. (*Hint:* Make an analogy to the case of a particle in a box with walls at $x = 0$ and $x = L$.)
12. A ruby laser emits light of wavelength 694.3 nm. If this light is due to transitions from the $n = 2$ state to the $n = 1$ state of an electron in a box, find the width of the box.
13. A proton is confined to moving in a one-dimensional box of width 0.200 nm. (a) Find the lowest possible energy of the proton. (b) What is the lowest possible energy of an electron confined to the same box? (c) How do you account for the large difference in your results for (a) and (b)?
14. A particle of mass m is placed in a one-dimensional box of length L. The box is so small that the particle's motion is *relativistic,* so that $E = p^2/2m$ is *not valid.* (a) Derive an expression for the energy levels of the particle using the relativistic energy–momentum relation and the quantization of momentum that derives from confinement. (b) If the particle is an electron in a box of length $L = 1.00 \times 10^{-12}$ m, find its lowest possible kinetic energy. By what percent is the nonrelativistic formula for the energy in error?
15. Consider a "crystal" consisting of two nuclei and two electrons, as shown in Figure P6.15. (a) Taking into account all the pairs of interactions, find the potential energy of the system as a function of d. (b) Assuming the electrons to be restricted to a one-dimensional box of length $3d$, find the minimum kinetic energy of the two electrons. (c) Find the value of d for which the total energy is a *minimum.* (d) Compare this value of d with the spacing of atoms in lithium, which has a density of 0.53 g/cm^3 and an atomic weight of 7. (This type of calculation can be used to estimate the densities of crystals and certain stars.)

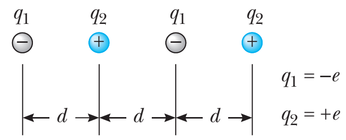

Figure P6.15

16. An electron is trapped in an infinitely deep potential well 0.300 nm in width. (a) If the electron is in its ground state, what is the probability of finding it within 0.100 nm of the left-hand wall? (b) Repeat (a) for an electron in the 99th excited state ($n = 100$). (c) Are your answers consistent with the correspondence principle?
17. An electron is trapped at a defect in a crystal. The defect may be modeled as a one-dimensional, rigid-walled box of width 1.00 nm. (a) Sketch the wavefunctions and probability densities for the $n = 1$ and $n = 2$ states. (b) For the $n = 1$ state, find the probability of finding the electron between $x_1 = 0.15$ nm and $x_2 = 0.35$ nm, where $x = 0$ is the left side of the box. (c) Repeat (b) for the $n = 2$ state. (d) Calculate the energies in electron volts of the $n = 1$ and $n = 2$ states.
18. Find the points of maximum and minimum probability density for the nth state of a particle in a one-dimensional box. Check your result for the $n = 2$ state.
19. A 1.00-g marble is constrained to roll inside a tube of length $L = 1.00$ cm. The tube is capped at both ends. Modeling this as a one-dimensional infinite square well, find the value of the quantum number n if the marble is initially given an energy of 1.00 mJ. Calculate the *excitation energy* required to promote the marble to the next available energy state.

6.5 The Finite Square Well

20. Consider a particle with energy E bound to a *finite* square well of height U and width $2L$ situated on $-L \leq x \leq +L$. Because the potential energy is symmetric about the midpoint of the well, the stationary state waves will be either symmetric or antisymmetric about this point. (a) Show that for $E < U$, the conditions for smooth joining of the interior and exterior waves lead to the following equation for the allowed energies of the symmetric waves:

$$k \tan kL = \alpha \qquad \text{(symmetric case)}$$